PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Evolutive Spectral Analysis of Sunspot Data over the past 300 Years [and Discussion]

A. Berger, J. L. Melice, I. Van Der Mersch, M. Beran, A. Provenzale and J. L. Stanford

Phil. Trans. R. Soc. Lond. A 1990 **330**, 529-541
doi: 10.1098/rsta.1990.0034

Phil. Trans. R. Soc. Lond. A **330**, 529–541 (1990)
Printed in Great Britain

Evolutive spectral analysis of sunspot data over the past 300 years

By A. Berger, J. L. Mélice and I. van der Mersch

Université Catholique de Louvain, Institut d'Astronomie et de Géophysique Georges Lemaître, chemin du Cyclotron 2, *B*-1348 *Louvain-la-Neuve, Belgium*

We analyse the series of the Wolf sunspot number in the frequency domain to determine the dimension of the solar cycle system by using the properties of its strange attractor and to study the stability in time of this dimensionality and of the main quasi-periodicities.

The two classical methods of time series analysis, Fourier harmonic and Blackman–Tukey spectral analysis, have been applied first to the series of the annual Wolf sunspot numbers to determine its overall character. To detect stationarity, periodic regression based upon the three most statistically significant quasi-periods and especially a moving form of the maximum entropy spectrum analysis (MESA) have been used. Both analyses show a splitting of the 11-year cycle before 1800, when a ±55-year cycle is dominant, and a single 11-year and ±100-year peak after 1800. Moreover, these quasi-periods are very sensitive to the time interval over which the analysis is carried out. The reason is that the sunspot numbers constitute a widely non-stationary process, which therefore implies that Fourier techniques are not useful to predict solar activity and must be used as fitting procedure only.

The minimum cross-entropy method serves to improve the maximum entropy spectrum. With a good *a priori* estimate and data containing a low noise level, this method allows the detection of very close peaks and the refinement of the main frequencies; it does not split nor introduce artificial peaks. The Thomson model was also applied for its superior bias control, its excellent leakage resistance and a better statistical information.

The same methods were then used to study the 22-year magnetic cycle, which is formed by taking into account the change in polarity of the succeeding 11-year cycle. The moving form of MESA confirms the 22-year cycle to be highly stable in contrast to the instability in the period of the 11-year sunspot series. This suggests the importance of working with the more invariant 22-year magnetic series to explain the complex, non-stationary behaviour of the sunspot series and of the solar–terrestrial interactions.

Finally, we tried to see if the system generated by the sunspot data was allowing the existence of an attractor and tried to determine the minimum number of variables necessary to describe this system. It is shown that the dimension of the attractor is highly unstable varying from 2.21 to 4.95 in a quasi-cyclic way.

INTRODUCTION

Attempts to analyse the time series of Wolf sunspot numbers have a long history dating back to the work of Yule (1927). With the advent of powerful computers and the construction of modern efficient techniques of data analysis in the frequency domain, many authors have attempted to understand better its spectral structure (Cohen & Lintz 1974; Sonett 1984; Sneyers & Cugnon 1986).

When reviewing all these results, one is puzzled by the lack of predictability of the future sunspot numbers using just statistical treatment of the sunspot numbers alone without reference to any external predictor. Moreover, the periods found in these statistical investigations are different. The question that naturally arises, besides recognizing that physical models of solar activity are most probably the key to the problem, are therefore:

(i) are these results significantly different from each other?

(ii) are these differences due to the nature of the techniques used and/or to the instability of the time series itself, all the time intervals investigated being different?

The goal of this study is precisely to try to answer these questions by analysing the spectral characteristics of the sunspot numbers time series over a prescribed interval of time by using six different spectrum models, namely, periodic regression (Bloomfield 1976), harmonic analysis, Blackman–Tukey spectrum analysis (Jenkins & Watts 1968), Thomson (multitaper) spectrum analysis (Thomson 1982), maximum entropy spectrum analysis (Burg 1972) and minimum cross-entropy spectrum analysis (Shore 1981). Our results are not expected to be fundamentally different from others but our approach, comparing the results obtained from different techniques and different intervals of time, aims to clarify the problem.

SPECTRA OF THE 1700–1986 SUNSPOT TIME SERIES

To study the general characteristics of the annual Wolf sunspot numbers, the classical fitting procedure of harmonic analysis was first applied to the observations. Figure 1 shows this harmonic analysis of the whole sunspot series (1700–1986); a total of 27 harmonics (all above the 80% confidence level) are needed to explain 90% of the total variance. The most important peaks appear around 11, 96 and 57 years.

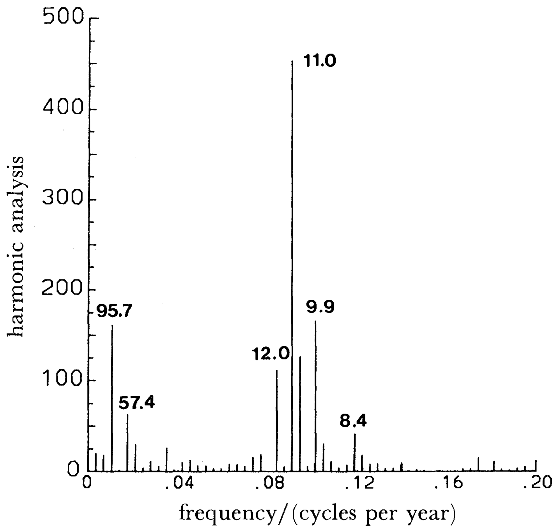

FIGURE 1. Harmonic analysis of the annual Wolf's sunspot number from 1700 to 1986.

It is interesting to note that these harmonics reach their maxima close to 1957, which contributes to the high sunspot numbers of 1957–58. A periodic regression using the 27 most significant components has also shown a good agreement with the results obtained by others provided the same interval of time is used, which demonstrates that the frequencies found in both models are independent of the techniques.

To improve the statistical behaviour and stability of the above detected spectral estimates, the same data-set was then analysed by using 'Blackman–Tukey' (BT) spectrum analysis (figure 2) for a maximum lag of 96 years, which corresponds to one-third of the total length of the series. As the first three computed autocorrelation coefficients of the series show that the analysed record does not behave as a Markov process, a 'white noise' continuum was chosen as a null hypothesis. The 97.5 % upper confidence limit at the 0.05 significance level drawn on figure 2 shows that only the 11-year period is significant. In addition, a weak non-significant 100-year peak is also found.

Thomson's 'multitaper' spectrum analysis was next performed to the same series for its high-frequency resolution and its amplitude estimate properties. As seen in figure 3, the significant 11-year period of the BT analysis splits into two peaks with a corresponding period fluctuating between 11 and 10 years. These results compare well with those obtained from the harmonic analysis. They show a clear non-stationarity of the frequency of the 11-year oscillation. In the low-frequency part of the spectrum, the broad 100-year period of the BT analysis splits also in two peaks with a period of respectively 100 and 56 years, again closely related to the harmonics 3 and 5 of the harmonic analysis.

FIGURE 2

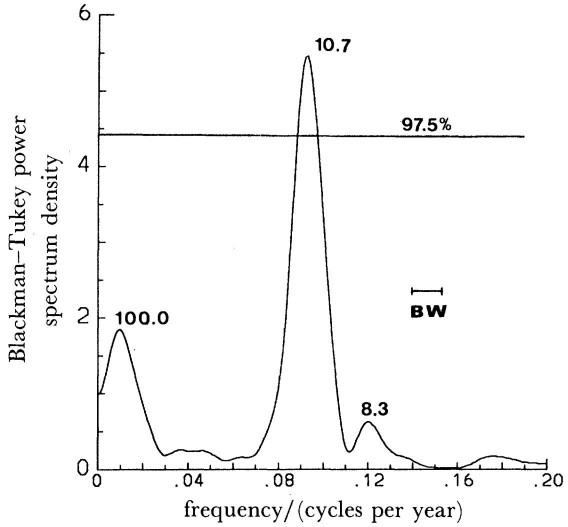

FIGURE 3

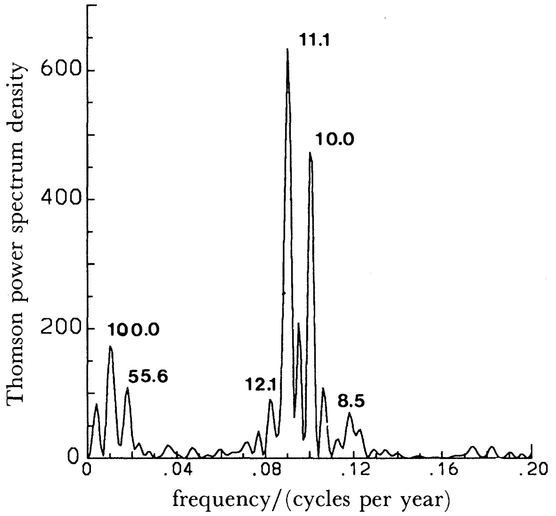

FIGURE 2. Blackman–Tukey spectrum analysis of the annual Wolf's sunspot number from 1700 to 1986. The bandwidth and the 97.5 % upper confidence limit for a 'white noise' continuum are also provided.

FIGURE 3. Thomson 'multitaper' spectrum analysis of the annual Wolf's sunspot number from 1700 to 1986.

To improve the frequency resolution for trying to solve this problem of splitted peaks, the maximum entropy spectrum analysis (MESA) technique was applied. It allows also to investigate the low-frequency part of the spectrum in much greater detail. Remarkably, as in the Thomson analysis, the 11-year oscillation becomes a broad bimodal peak with two main periods of 11.1 and 10 years (figure 4). This definitely indicates that the 11-year quasi-periodicity has slightly changed with time. The 100 and around 52–56-year peaks are also clearly present in the low-frequency part of the spectrum.

Finally, a minimum cross-entropy spectrum analysis performed to the same data-set is presented in figure 5. The advantage of this method compared with MESA is its ability to detect very close spectral peaks with a higher accuracy. The results obtained, compared with those given by Thomson and MESA analyses, confirm definitely the non-stationarity of the 11-year quasi-periodicity.

FIGURE 4

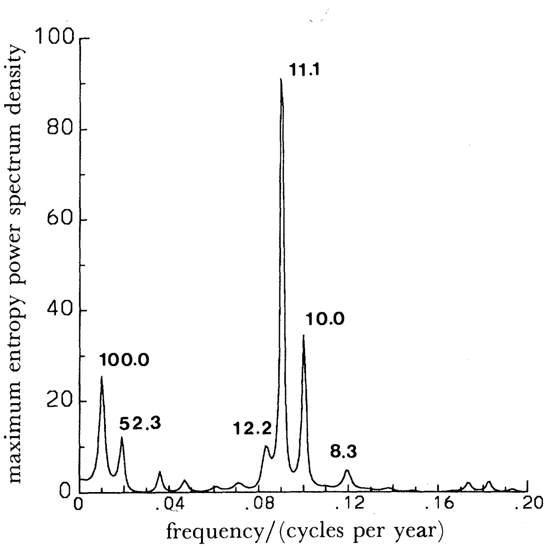

FIGURE 5

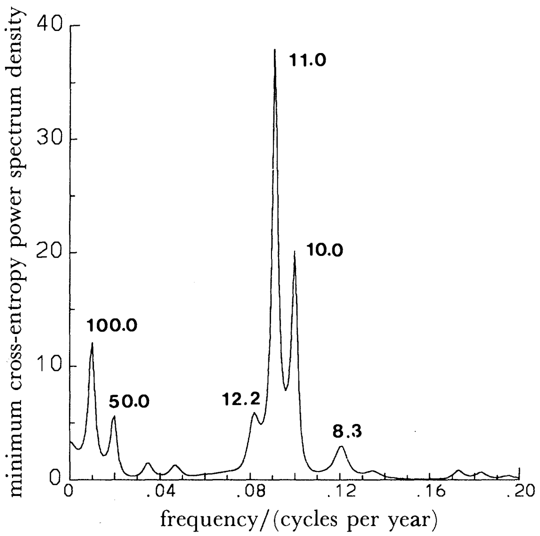

FIGURE 4. Maximum entropy spectrum analysis of the annual Wolf's sunspot number from 1700 to 1986.

FIGURE 5. Minimum cross-entropy spectrum analysis of the annual Wolf's sunspot number from 1700 to 1986.

NON-STATIONARITY OF THE WOLF SUNSPOT NUMBERS

An instability in frequency was thus broadly diagnosed with the above performed spectral analyses. From a simple plot of the Wolf number, it is also obvious that the cycles of activity vary in amplitude, the cycles of higher activity having a maximum Wolf number that can be four times as large as the cycle of lowest activity. Thus the question arises, as to how to resolve this instability in time of both the amplitudes and frequencies. Periodic regression and evolutive MESA will be used together with an evolutive Thomson analysis and complex demodulation to answer this question.

A moving form of MESA (Radoski *et al.* 1976) was used to answer this question of stability. The spectrum of the series made of the first 70 years is computed. Then this 'data-window' is moved forward 10 years and the next spectrum is computed for 1710–1780 and so on. This moving spectral analysis gives an overview of the shift or the persistence of the spectral peaks with time. The three-dimensional plot of these figures both in perspective (figure 6) and from the front (figure 7) gives the spectral density as a function of frequency for the 22 different 70-year subperiods analysed. In addition to the variation of the amplitude of some stationary peaks, note the large shift in the frequency of the 11-year quasi-period that indeed varies between 8 and 14 years.

As the maximum entropy amplitude estimates have to be regarded with caution, an evolutive Thomson analysis was performed and presented in figure 8 in a similar way, to give a more precise idea of the amplitude variation of all the quasi-periods. For 1700–1810, we observed a single peak around 58 years and a split in two peaks of the 11-year cycle around 14 and 8 years. From 1810 to 1950, the 58-year peak has disappeared and the 11-year periodicity becomes more stable in frequency. A very weak five-year peak appears also

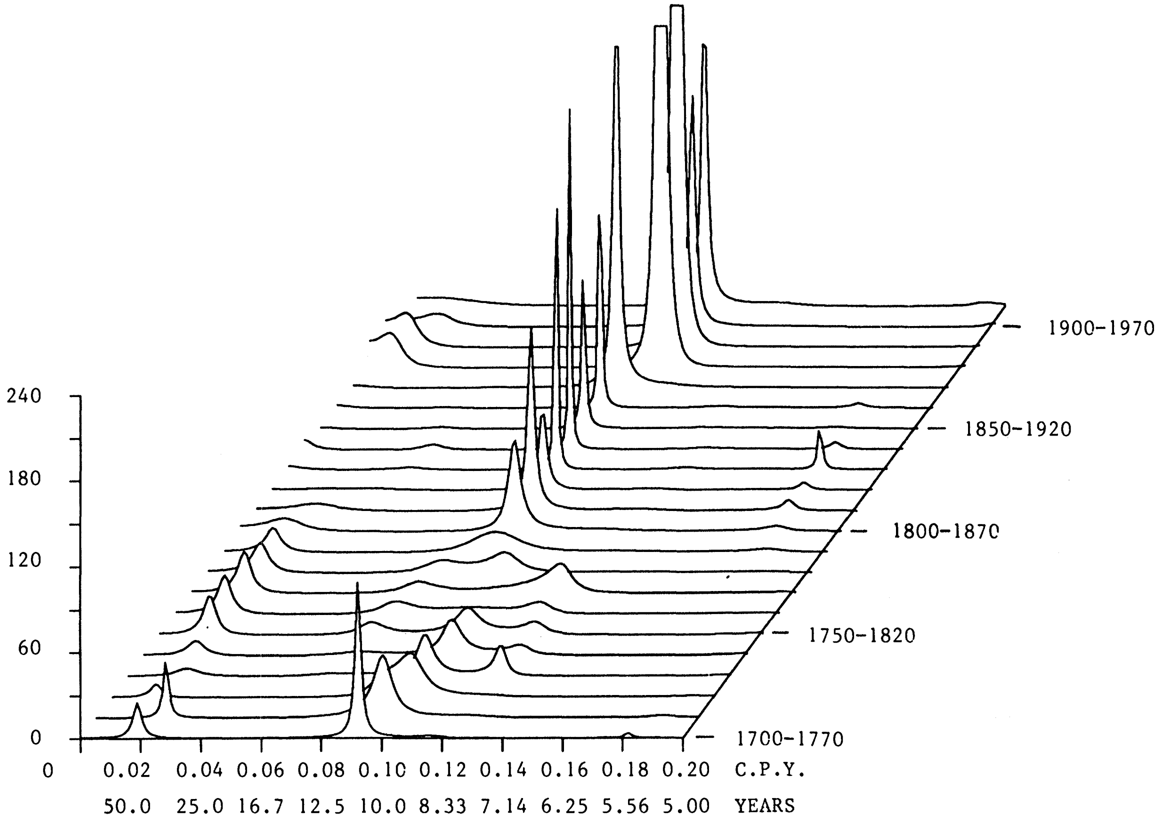

FIGURE 6. Evolutive maximum entropy spectrum analysis of the annual Wolf's sunspot number from 1700 to 1986. Note that the quasi 11-year peaks of the spectra corresponding to the series 1870–1940, 1880–1950, 1890–1960 and 1910–80 are truncated.

sometimes during the same interval. The instability of the 11-year cycle thus tends to occur when the 58-year peak is present and its amplitude is low during this interval of time (1700–1810).

THE 22-YEAR MAGNETIC SERIES

Transformation of the sunspot series can be carried out by considering the reversal of the sign between the alternating half magnetic cycles. This transformed series has a much stronger physical basis as it accounts not only for the reversal of the magnetism of the leading spots in alternate 11-year cycles, but also for some variation in the length and amplitude of this 11-year cycle. Indeed, there is a tendency for long and short cycles to alternate but mainly there is a clear antisymmetry between the period of increasing sunspot numbers and the period of decreasing values. The duration of the magnetic cycle, which combines successive 11-year cycles, should thus be more stable than that of the 11-year cycle.

Harmonic analysis, Blackman–Tukey, Thomson, maximum entropy and minimum cross-entropy spectral analysis were performed to this 22-year magnetic series from 1700 to 1986 by using the same parameters as for the preceding spectral analyses. The MESA is given in figure 9 (but all the other technics give the same result). As it can be seen, the main statistical advantage of this magnetic version of the Wolf sunspot number time series is the high stability

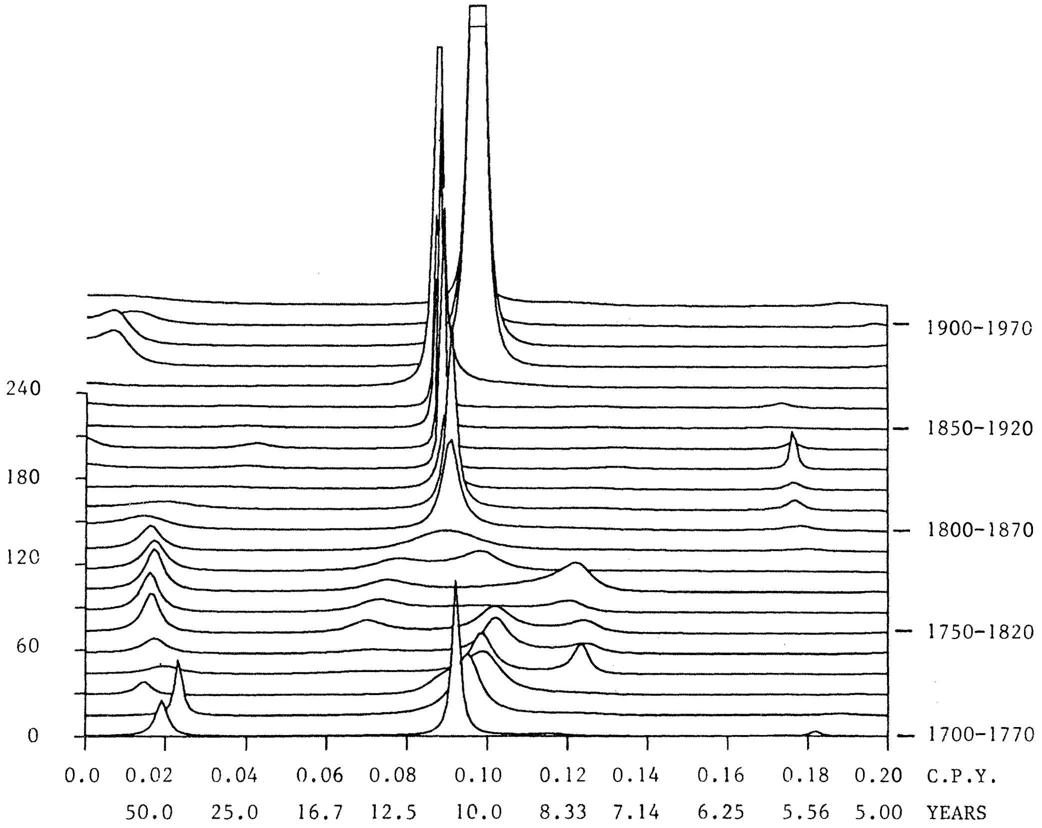

FIGURE 7. As figure 6.

of the 22-year cycle which explains 62 % of the total variance over the period 1700–1986 in contrast to the frequency instability of 11-year sunspot amplitude time series.

To demonstrate this higher stability in frequency of the 22-year cycle, an evolutive MESA (figures 10 and 11) and evolutive Thomson analysis (figure 12) were calculated based on sub-series having a length of 110 years and a time shift of 10 years. The results confirm this stability of the 22-year cycle.

The evolutionary MESA does not show any difference between early and late data: the 1700–1810 data window and the subsequent nineteenth- and twentieth-century windows are quite similar; i.e. there is no evidence that the early data are actually different from the later data, on a magnetic basis, which is also shown by the evolutive Thomson analysis

This result has interesting implications for eliciting physical mechanisms. It shows the existence of an invariant structure in what is otherwise complex and unstable data and suggests the importance of working with the 22-year magnetic series to provide reliable extrapolative patterns. Moreover, the change in the 11-year amplitude cycle after 1800 could suggest a complex internal 'oscillation' within the basically quite stable 22-year magnetic cycle instead of the inadequacy of early data.

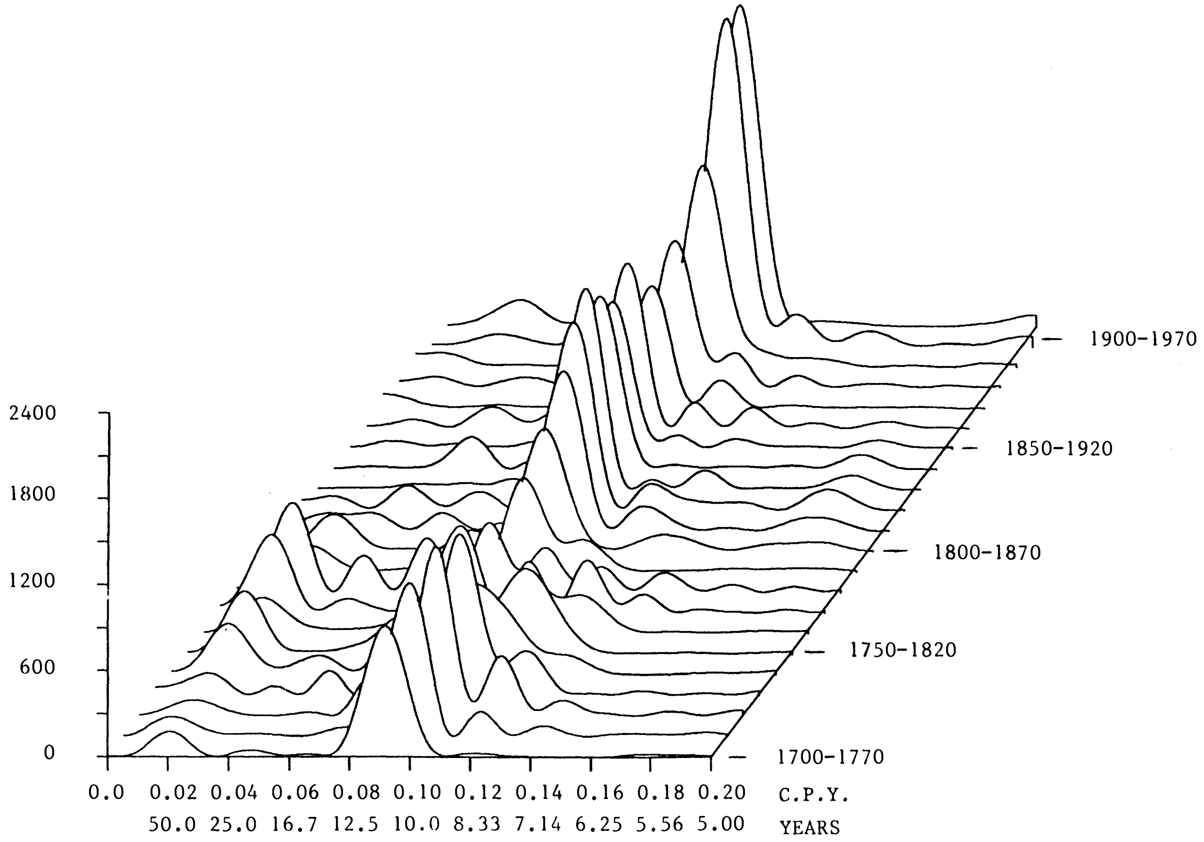

Figure 8. Evolutive Thomson 'multitaper' spectrum analysis of the annual Wolf's sunspot number from 1700 to 1986.

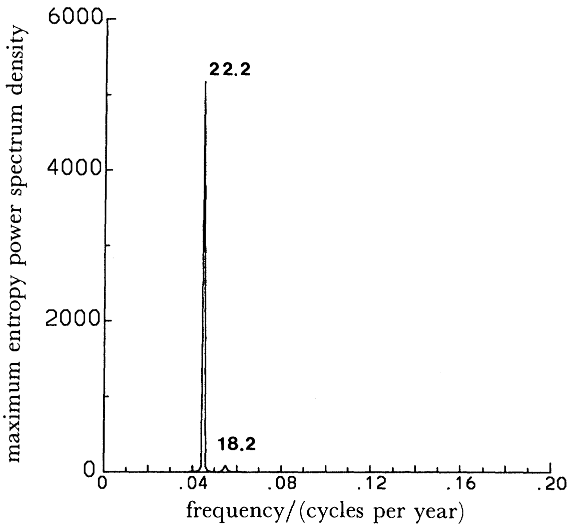

Figure 9. Maximum entropy spectrum analysis of the annual 22-year magnetic sunspot series from 1700 to 1986.

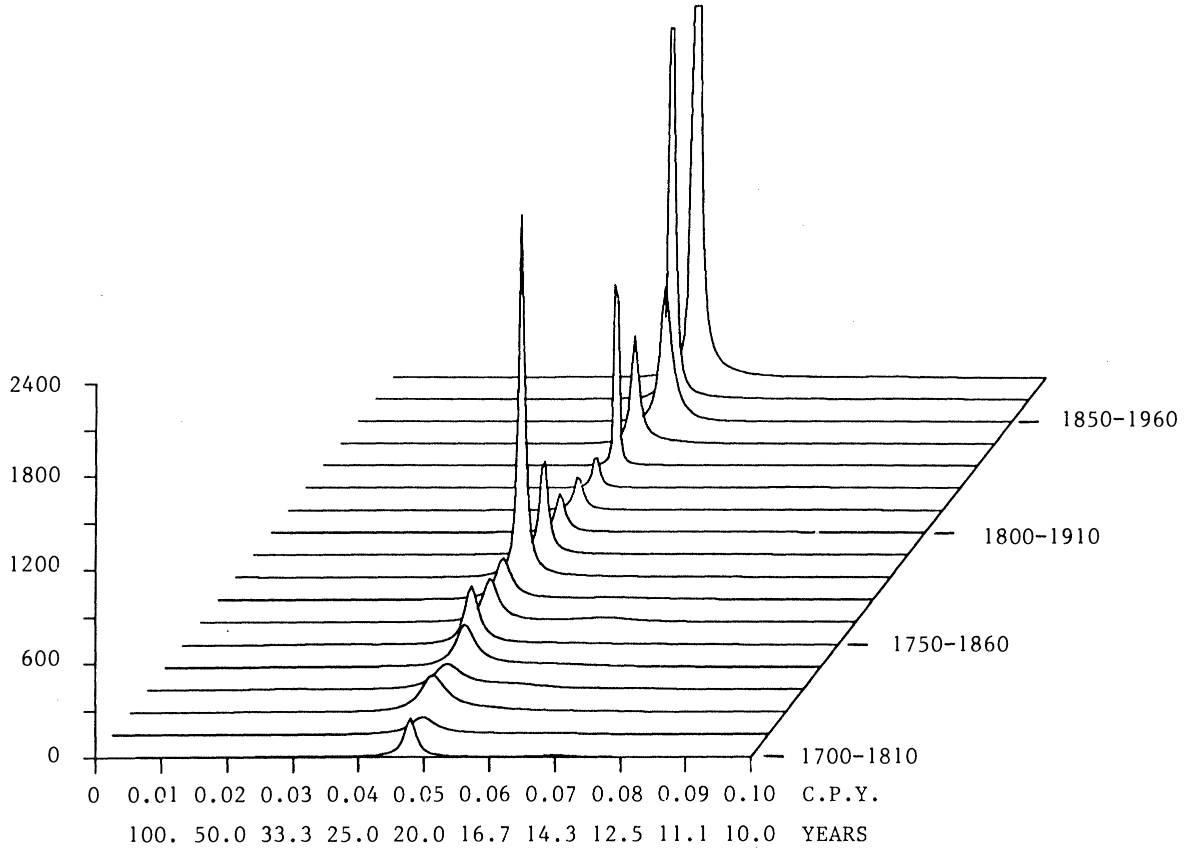

FIGURE 10. Evolutive maximum entropy spectrum analysis of the annual 22-year magnetic sunspot series from 1700 to 1986. Note that the quasi 22-year peaks of the spectra corresponding to the series 1860–1970 and 1870–1980 are truncated.

COMPLEX DEMODULATION

Complex demodulation (Bloomfield 1976) is a useful technique to describe the non-stability in frequency and amplitude of quasi-periodic data such as the Wolf sunspot number series. The amplitude curve ((*a*) in figure 13) shows that there are indeed substantial variations in amplitude of the 11-year quasi-period with a range of about 3:1.

The phase curve ((*a*) in figure 14) also displays substantial variations. These variations of the instantaneous phase may be interpreted in terms of a corresponding shift of the frequency around the 11-year quasi-oscillation. As an example, the increase in instantaneous phase between 1763 and 1790 compared with the preceding time interval 1724–62 corresponds to a decrease in period of the 11-year quasi-period, whereas the following instantaneous phase drop between 1790 and 1829 corresponds to an increase in the period of this oscillation.

Complex demodulation was then applied to the magnetic 22-year sunspot series. The corresponding results are given in figures 13 and 14 (curves (*b*)). As expected, the instantaneous phase shows a higher stability in frequency of the 22-year oscillation when compared with the highly unstable 11-year quasi-periodicity but the instantaneous amplitude variation of the 22-year magnetic curve is comparable with the 11-year amplitude variation.

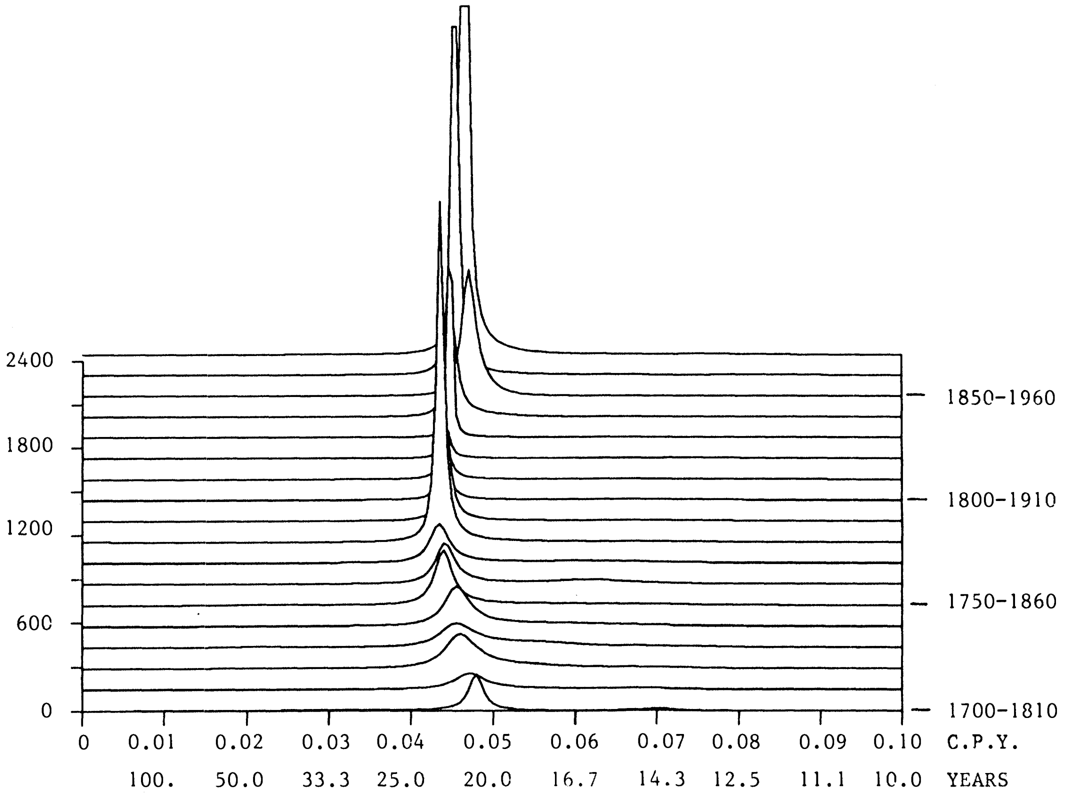

Figure 11. As figure 10.

Dimension of the strange attractor of the solar cycle

Finally, we try to see if the system generated by the sunspot data allows the existence of an attractor and to determine the minimum number of variables necessary to describe this system. Our data will be the monthly sunspot numbers from 1750 to 1986. In figure 15, $C(r)$ is the logarithm of the correlation function of the attractor and r is the radius of the ball around any point of the phase space defined from the lagged variables of the time series. The saturation value of the slope $C(r)$ against r is here 3.15 which means that four variables should be necessary to describe the behaviour of the sunspot system. However, an evolutive version of this technique used for successive 40-year-long intervals, shows that this dimension of the attractor is highly unstable varying from 2.21 to 4.95 in a quasi-cyclic way.

Conclusions

This study applying six different spectral techniques to the Wolf sunspot time series confirms that the physical process producing sunspots is not stationary as already pointed out by Eddy (1977). The spectra indicate that the whole time series (1700–1986) is characterized by the existence of three main peaks which periods are ±100, ±55 and ±11 years. The 11-year quasi-period is, however, the only one to be statistically significant following the Blackman–Tukey criteria. Moreover, it has two main components which are 11.1 and 10 years.

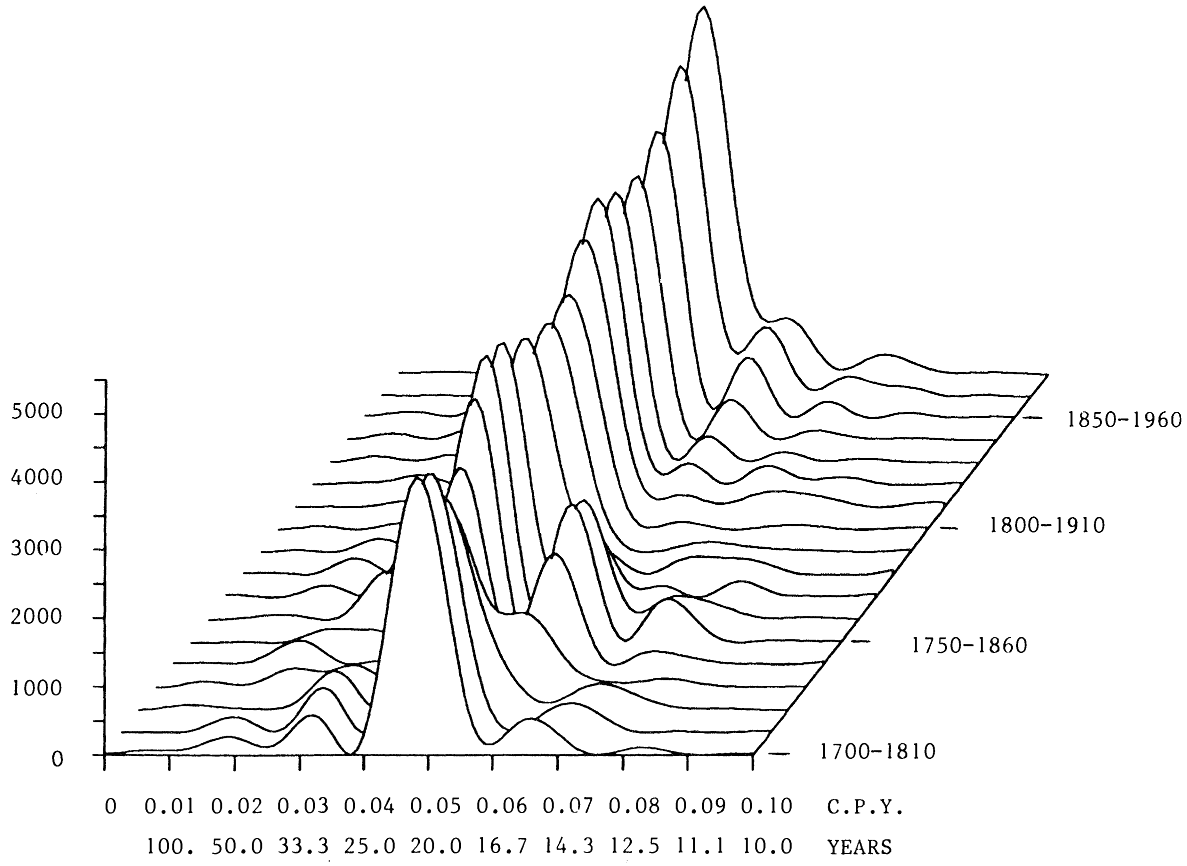

FIGURE 12. Evolutive Thomson 'multitaper' spectrum analysis of the annual 22-year magnetic sunspot series from 1700 to 1986.

FIGURE 13

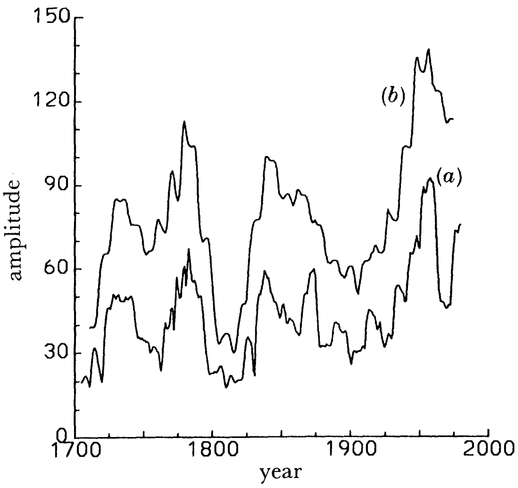

FIGURE 14

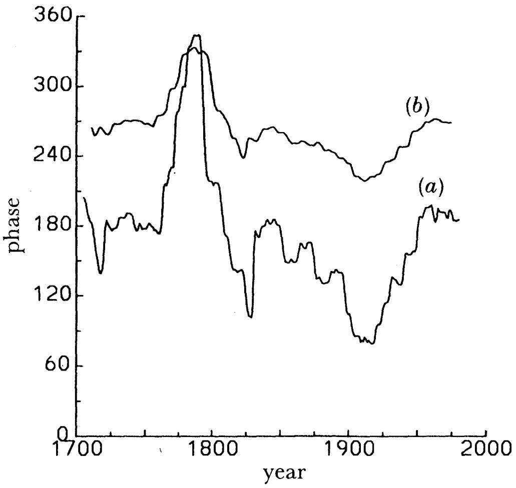

FIGURE 13. Instantaneous amplitude of the annual Wolf's sunspot number (*a*) and of the annual 22-year magnetic sunspot series (*b*) from 1700 to 1986 (smoothed by taking a simple moving average of respectively 11 and 22 years).

FIGURE 14. Instantaneous phase of the annual Wolf's sunspot number (*a*) and of the annual 22-year magnetic sunspot series (*b*) from 1700 to 1986 (smoothed by taking a simple moving average of respectively 11 and 22 years).

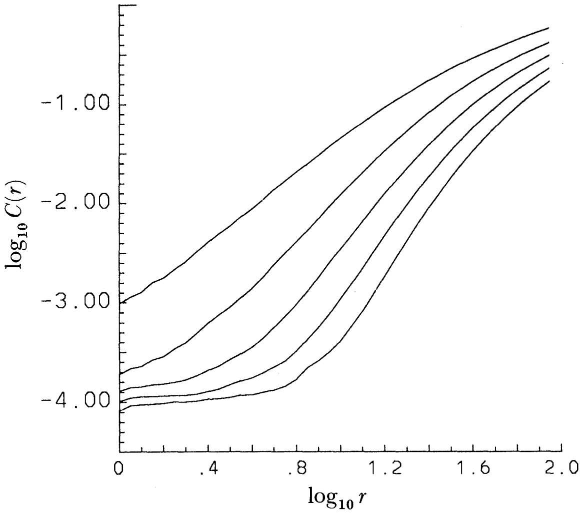

FIGURE 15. $\log_{10} C(r)$ against $\log_{10} r$ for the monthly sunspot data from 1750 to 1986. The curves are computed for $n = 2$ (first left curve) until $n = 6$ (last right curve).

This bimodal characteristic of the 11-year cycle might also be interesting in the frame of the existence of a long period ondulation (*ca.* 176–180 years), which is usually suggested by the planetary theory of sunspots and which is a beat phenomenon not caused by a primary long-term excitation function. The best correlation occurs at frequencies that correspond to periods of 11.2 and 9.9 years, but numerical values found in the literature for different time intervals may generate a long-period ranging from 167 to 220 years. Finally, we think that the sensitivity of this long-term period to the accuracy of the double 11-year peak and its potential meaning in terms of the physical origin of the sunspots and of the impact of the solar activity on climate justifies this kind of a better determination of the statistical properties of this Wolf sunspot time series.

We are grateful to Professor R. D. Rosen, AER, Cambridge (Massachusetts, U.S.A.) for his critical remarks on an early version of this paper, and to Dr A. Koeckelenbergh, Belgian Royal Observatory, Brussels, and Sunspot Index Data Center of the Federation of Astronomical and Geophysical Data Analysis Services (funded by the International Council of Scientific Unions under the auspices of the International Union of Geodesy and Geophysics, International Astronomical Union and the Union Radioscientifique Internationale). We also thank Dr C. Nicolis (Institute of Space Aeronomy, Brussels, Belgium) for providing us with her program on the strange attractor.

REFERENCES

Bloomfield, P. 1976 *Fourier analysis of time series: an introduction*. New York: Wiley.

Burg, J. P. 1972 The relationship between maximum entropy spectra and maximum like-hood spectra. *Geophys.* **37**, 375–376.

Cohen, T. J. & Lintz, P. R. 1974 Long-term periodicities in the sunspot cycle. *Nature, Lond.* **250**, 398–399.

Eddy, J. A. 1977 Climate and the changing sun. *Climatic Change* **1**(2), 173–190.

Jenkins, G. & Watts, D. 1968 *Spectral analysis and its applications*. San Francisco: Holden-Day.

Radoski, H. R., Zawalich, E. J. & Fougere, P. F. 1976 The superiority of maximum entropy power spectrum techniques applied to geomagnetic micropulsations. *Phys. Earth planet. Inter.* **12**, 208–216.

Shore, J. L. 1981 Minimum cross-entropy analysis. *IEEE Trans. Acoust. Speech signal processing*, **ASSP-29**, 230–237.

Sneyers, R. & Cugnon, P. 1986 On the predictability of the Wolf sunspot number. *Annls. Geophysicae* **4**, 81–86.
Sonett, C. P. 1984 Very long solar periods and the radiocarbon record. *Rev. Geophys. Space Phys.* **22**, 239–254.
Thomson, D. J. 1982 Spectrum estimation and harmonic analysis. *IEEE Proc.* **70**, 1055–1096.
Yule, U. G. 1927 On a method of investigating periodicities in disturbed series, with a special reference to Wolfer's sunspot numbers. *Phil. Trans. R. Soc. Lond.* A **226**, 267–298.

Discussion

M. Beran (*Institute of Hydrology, Wallingford, U. K.*). I note the somewhat arbitrary and subjective definition of the Wolf sunspot number. Is Professor Berger concerned that he may be applying his numerical analysis to what looks like the results of a Rorschach ink blot test?

A. Berger. An evolutive maximum entropy spectral analysis (MESA) of the yearly sunspot data series from 1700 to 1986 had indeed indicated that the 11-year cycle is particularly unstable during the early part of the record (before 1810). At that time, a ±55-year peak is more significant. This difference between the MESA results for the more recent observations and the early ones could suggest that the early sunspot data must be treated with care.

However, the splitting in the pre-1800 period of the 11-year peak could be linked to the combination of the three most important harmonics instead of just suggesting early data inadequacy. Moreover, the high-frequency stability of the 22-year cycle through the whole time span (1750–1986) seems to support the reliability of these data, at least for the magnetic series.

These results show thus clearly that adequate statistical analyses of *a priori* non-reliable data is a worth while exercise in the sense that these analyses may question the data and the validity of the non-reliability assumption in an objective manner.

A. Provenzale (*Instituto di Cosmogeofisica, Torino, Italy*). Are the fluctuations (in amplitude and frequency) of the 11-year spectral peak in sunspot dates statistically signifiant? I caution the attractor dimension calculations because (*a*) the number of points in the sunspot number can be insufficient for the dimension evaluation (see Smith 1988); (*b*) I don't see a clean scaling range in the correlation curves Professor Berger has shown; (*c*) the calculation of a finite spectral dimension does not imply the existence of low-dimensional chaos, because there are simple stochastic processes with finite fractal dimension (see Osborne & Provenzale 1989).

A. Berger. Harmonic analysis of the yearly sunspot data from 1700 to 1986 shows that for the 11 harmonics that are detected in the interval between 11.96 and 8.44 years, five are significant at the 0.01 significance level (with periods of respectively 11.96, 11.04, 10.63, 9.90 and 8.44 years) and one is significant at the 0.02 significance level (with a period of 9.57 years). This shows clearly that the splitting of the 11-year peak is not only a result of statistical fluctuations.

J. L. Stanford (*Department of Atmospheric, Oceanic and Planetary Physics, University of Oxford, U.K.*). Dr Ribes (this Symposium) discusses solar diameter oscillations with periods near that of the quasi-biennial oscillation (QBO), and Dr Labitzke (this Symposium) discusses apparent correlations between sunspots and atmospheric phenomena at similar periods. Has Professor Berger performed spectral analyses with sunspot time series spaced at less than yearly intervals, and if so, does he find a statistically significant signal near QBO periods (slightly more than two years)?

A. Berger. We have indeed performed harmonic analysis and Blackman–Tukey spectrum analysis of the monthly sunspot series from 1750 to 1986. When looking at the period interval from 24 to 36 months, the results are the following:

(1) no statistically significant signal at the 0.05 significance level in the interval 24 to 36 months;

(2) one harmonic with a period of 33.5 months detected as significant at the 0.1 significance level with the harmonic analysis;

(3) two harmonics with periods of respectively 25.86 and 25.62 months detected as significant at the 0.2 significance level with the harmonic analysis and which could be related to the QBO signal.

Z. Reut (*Bath, U.K.*). The dynamics of the solar system may have to be taken into account in explaining the variability of the Sun. The Sun revolves round the centre of mass of the Solar System that does not coincide with the Sun's own centre of mass due to the gravitational effect of planets (mainly Jupiter). The Sun's revolution relates to the 11-year solar cycle and to the 12-year period of the Jupiter's revolution round the Sun.

A. Berger. Indeed the Sun revolves round the centre of mass of the Solar System. This is taken into account in the computation of the planetary motion and in the related Milankovitch theory. This could play a role for simulating the physics of the Sun but according to Meeus (1975) the planetary tides on the Sun are negligible and the Jupiter effect does not exist.

Additional references

Meeus, J. 1975 *Icarus* **26**, 257–267.
Osborne, A. R. & Provenzale, A. 1989 Finite correlation dimension for stochastic systems with power-law spectra. *Physica* D**35**, 357–381.
Smith, A. L. 1988 *Phys. Lett.* A **133**, 283–288.